Chapter 11: Linking the Term Structures of Interest Rates and Macroeconomic Expectations – GS Yield Curve Valuation Model

We introduce a model of the US yield curve that is designed to provide regular trading signals. It is based on the idea that the term structure of interest rates is a reflection of the Fed's monetary policy stance and macroeconomic expectations at various horizons. Taking these economic variables as inputs, our model constructs a 'fair value' yield curve that, when compared with the actual curve, allows us to identify potential investment opportunities. We call this yield curve valuation model GS Curve.

Description of the Model

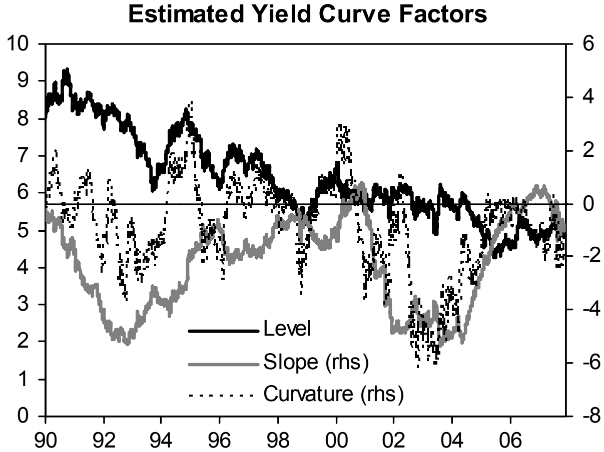
We model the whole term structure of government bond yields simultaneously by relating each point on the yield curve to a set of three independent factors – level, slope and curvature. Furthermore, instead of trying to explain the dynamics of yields at different maturities directly, we focus on describing the dynamics of those three underlying factors. Then, we are able to translate the dynamics of the three factors into the corresponding behaviour of yields at each tenor.

The chart below depicts the dynamics of the three factors extracted from the US yield curve over the period from January 1990 to November 2007.

The chart on the right shows that a three-factor approach gives a reasonably good fit of the actual yield curve. The ten charts on page 49 present a dynamic version of the same relationship, by plotting at different maturities the behaviour of actual yields against yields fitted using three factors.

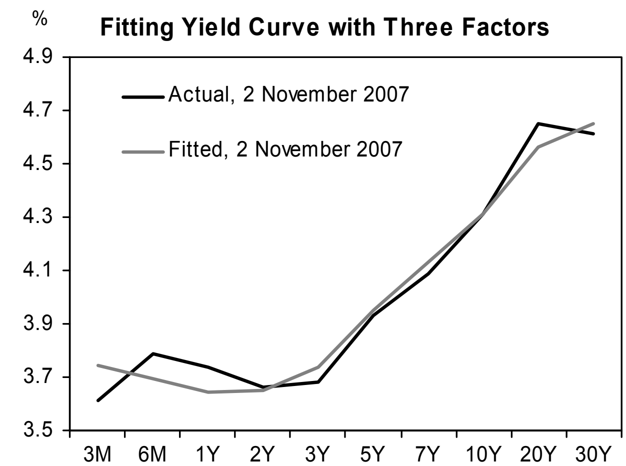
Next, we try to relate the three underlying factors to economic variables believed to be important determinants of interest rates in the economy. Specifically, we assume that the slope, curvature and level factors, which have the highest loadings in, respectively, the short-, medium- and long-term parts of the yield curve, are functions of, correspondingly, short (1-year-ahead), medium (3-years-ahead), and long-term (6-10-years-ahead) expectations of GDP growth (representing the real 'layer' of nominal interest rates) and CPI inflation (capturing the inflation 'overlay'). Additionally, the slope factor, being the determinant of the short-term part of the yield curve, also depends in our framework on the stance of monetary policy represented by the Fed funds rate.

Estimated Yield Curve Factors



Source: Goldman Sachs calculations.

Fitting Yield Curve with Three Factors

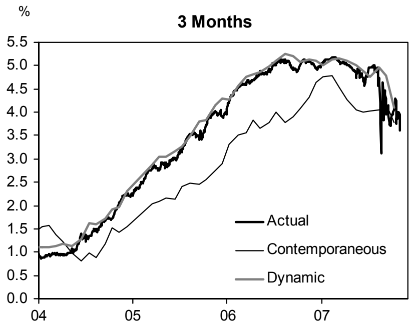


Sources: Haver Analytics, Goldman Sachs calculations.

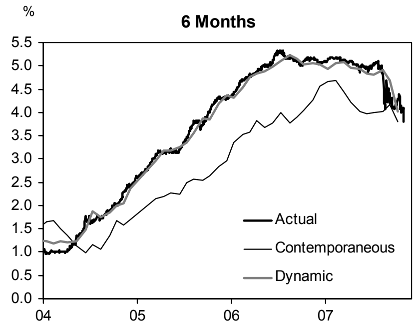
After estimating the historical relationship between the three yield curve factors and various economic variables, we are able to deduce what values of these three factors are implied by given economic fundamentals. Furthermore, we can translate these economically justified values of the three yield curve factors into the full term structure of bond yields, obtaining at the end a so-called 'fair value' yield curve.

When calculating the fair value yield curve implied by economic fundamentals, we use two slightly different methodologies. One approach entails using purely economic variables, ignoring past dynamics of yields, to construct a 'contemporaneous' fair value yield curve. Another approach recognises that interest rates may adjust to their fair values only gradually, and tries to account for this feature by controlling the values of the three yield curve factors for their own lags, giving rise to a 'dynamic' fair value.

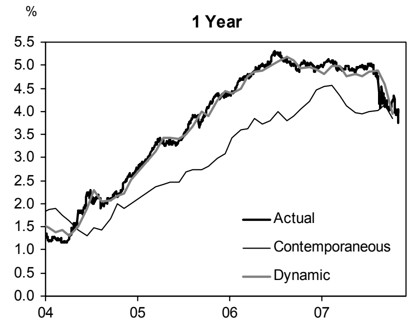
Our measures of macroeconomic expectations are based on Consensus Economics data. These data reflect the consensus forecasts of major macroeconomic variables collated by Consensus Economics during their regular surveys of market participants. Therefore, the fair values provided by the model are calculated using consensus expectations, not forecasts of Goldman Sachs economists.



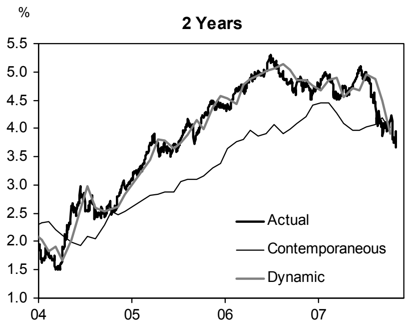
Sources: Haver Analytics, Goldman Sachs calculations.



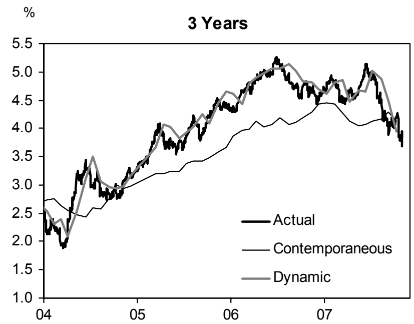
Sources: Haver Analytics, Goldman Sachs calculations.



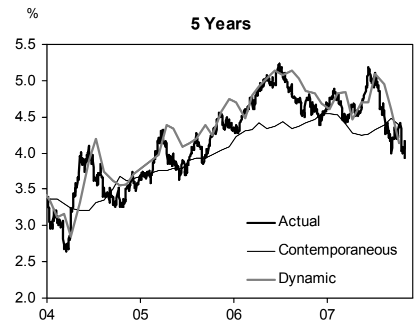
Sources: Haver Analytics, Goldman Sachs calculations.



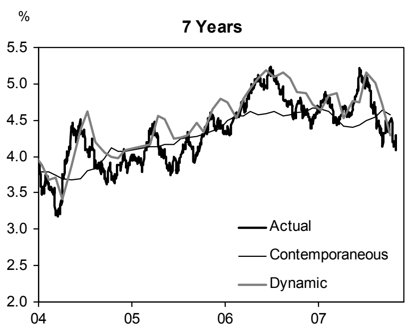
Sources: Haver Analytics, Goldman Sachs calculations.



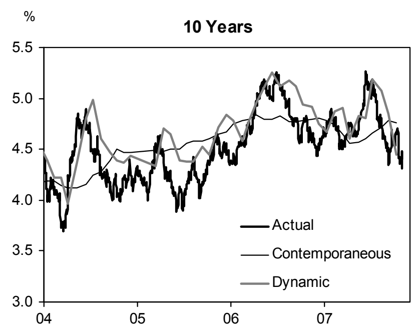
Sources: Haver Analytics, Goldman Sachs calculations.



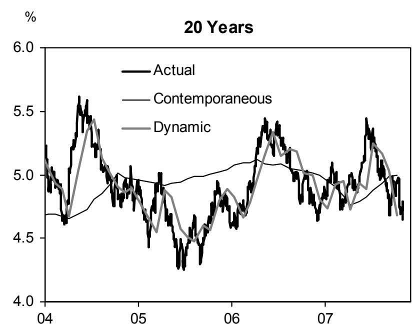
Sources: Haver Analytics, Goldman Sachs calculations.



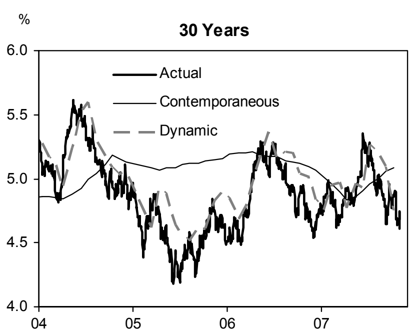
Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.

A more technical description of our methodology can be found in Boxes 1 and 2 at the end of the chapter.

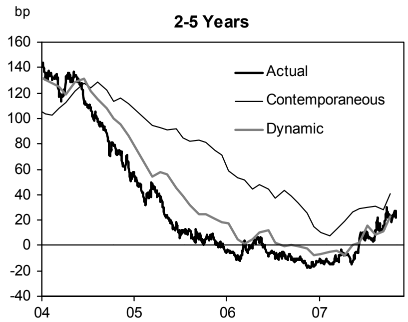
Model Output

The time series of different maturity yields juxtaposed against their contemporaneous and dynamic fair values for the dates between January 2004 and November 2007 are presented in the set of charts above. (The same charts for the full sample can be found on page 50.)

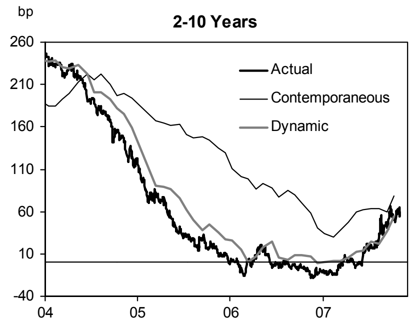
The two types of fair values have their own advantages and disadvantages. The advantage of the contemporaneous fair value is that the timing of its turning points often coincides with that of actual yields. Its main disadvantage stems from the well-known empirical fact that asset prices are more volatile than economic fundamentals – i.e., in our case the contemporaneous fair value does not explain a large portion of higher frequency movements in interest rates.

The advantage of the dynamic fair value is that, being as volatile as the actual yields are, it is better at recognising the higher frequency movements of interest rates that are not justified by changes in economic fundamentals. However, the behaviour of dynamic fair value often lags that of actual yields – a disadvantage stemming from the fact that dynamic fair value incorporates the mechanical dependence of the three yield curve factors on their own lags.

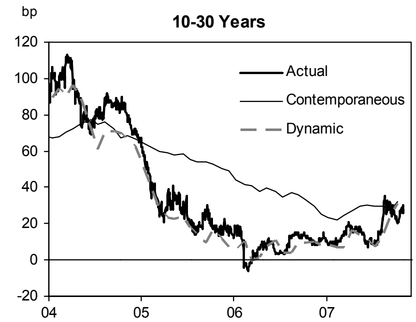
The model's output can also be viewed from a relativist perspective, which ignores the absolute levels of different maturity yields and focuses rather on various (linear, usually) combinations of those – i.e., spread products.



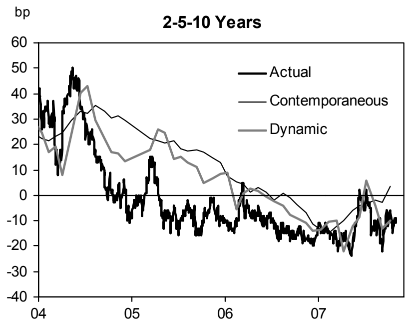
Sources: Haver Analytics, Goldman Sachs calculations.



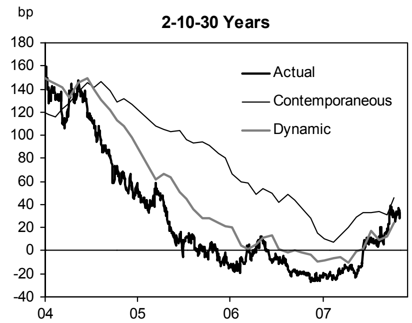
Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.



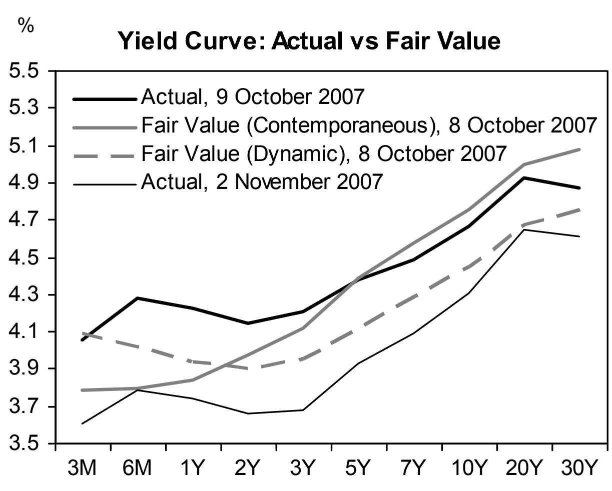
Sources: Haver Analytics, Goldman Sachs calculations.

The charts at top of the page present a selection of spreads that are popular among the practitioners: slopes (5-years less 2-years, 10-years less 2-years and 30-years less 10-years interest rates) as well as curvatures, also known as 'butterflies' (twice 5-years less 2-years and 10-years, as well as twice 10-years less 2-years and 30-years interest rates). (Similar charts for the full sample appear on page 51.)

Using the Model

Now we will demonstrate how the model can be used in formulating trading strategy. The chart to the right plots the actual term structure of interest rates against two types of fair values as of the beginning of October 2007. It also illustrates how the actual yields have moved since then. The same data are also presented in the table below (the misvaluation numbers can be interpreted as required yield changes if interest rates were to catch up with corresponding fair values, with the actually recorded yield changes given in the bottom line).

According to contemporaneous fair value, the yield curve was undervalued (i.e., actual yields were above fair ones) for maturities between three months and three years, fairly valued at a 5-years maturity, and overvalued (with actual yields below fair ones) for seven to 30-years maturities. In about one month's time, the actual curve has shifted downwards. We do not have the updated fair values available, but according to October contemporaneous fair value, the level shift was too aggressive. Interestingly, during this process, the better valued short end of the curve underperformed the less attractive long end. Turning to dynamic fair value, the yield curve was undervalued for all maturities except 3-months. This could help to explain the downward direction of the move in the yield curve over the month –



Sources: Haver Analytics, Goldman Sachs calculations.

Yield Curve: Misalignment and Subsequent Movement

	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
Misalignment (using dynamic fair value), 8-9 October 2007, bp	3	-26	-29	-25	-26	-25	-20	-22	-25	-11
Misalignment (using contemporaneous fair value), 8-9 October 2007, bp	-28	-48	-39	-18	-9	0	9	9	7	21
Yield change, 9 October-2 November 2007, bp	-45	-49	-49	-49	-53	-45	-40	-36	-28	-26

Source: Goldman Sachs calculations.

Yield Spreads: Misalignment and Subsequent Movement

	2-5Y	2-10Y	10-30Y	2-5-10Y	2-10-30Y
Misalignment (using dynamic fair value), 8-9 October 2007, bp	-1	3	11	-4	-8
Misalignment (using contemporaneous fair value), 8-9 October 2007, bp	18	26	12	10	14
Spread change, 9 October-2 November 2007, bp	4	13	10	-5	3

Source: Goldman Sachs calculations.

the curve in October appears to have overshot the levels that were observed in the near-term past (and which are reflected in dynamic fair value).

It is also possible to look at these results from the perspective of spread products discussed before. This is done in the table above (numbers here can be interpreted in the same way as in the previous table).

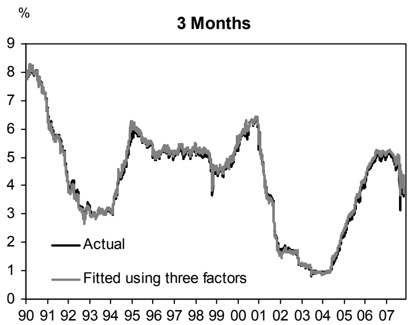
As can be seen, all the spreads we focus on were too high if compared to the contemporaneous fair value. For 2-10-years and 10-30-years slopes, the dynamic fair value was also pointing higher. Over the month, these two spreads moved up. The 2-5-years slope, however, was fair according to dynamic fair value. Hence, it is interesting to note that it did not steepen as much as other slopes did. The picture for butterflies was more mixed – their contemporaneous fair values were calling for higher spreads, while dynamic fair values were pointing in the opposite direction. Perhaps unsurprisingly then, these two spreads did not move much and ended the period considered roughly flat.

Conclusion

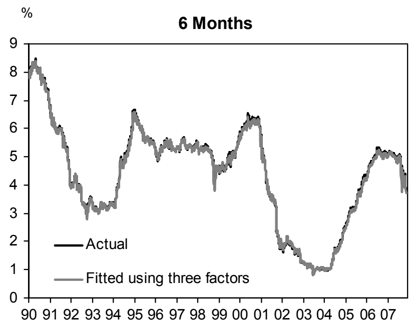
In principle, the output of the model can be used for valuation of the levels of specific tenor of interest rates, as well as of different spreads between selected tenors. However, the model seems to perform better in relative value than in directional interest rates space. Therefore, we prefer to apply it mostly for the formulation of our trading strategy in interest rates spreads.

Sergiy Verstyuk

Fitting The Term Structure of Interest Rates with Three Factors



Sources: Haver Analytics, Goldman Sachs calculations.



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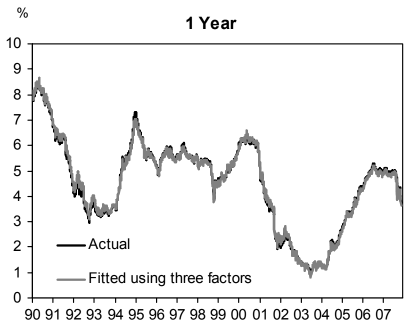

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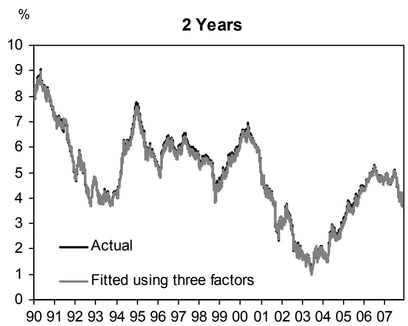

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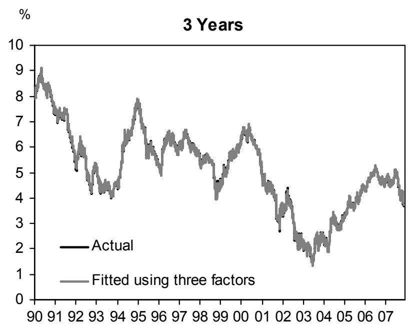

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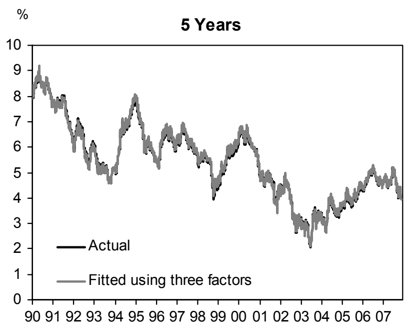

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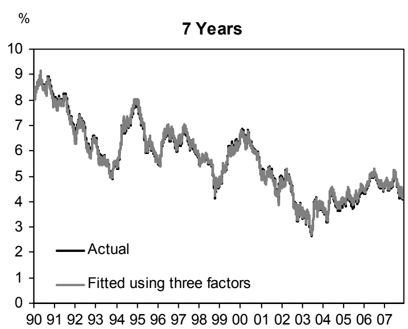
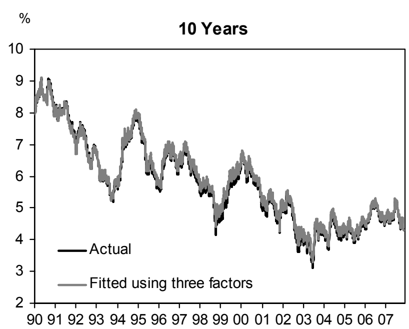


Sources: Haver Analytics, Goldman Sachs calculations.



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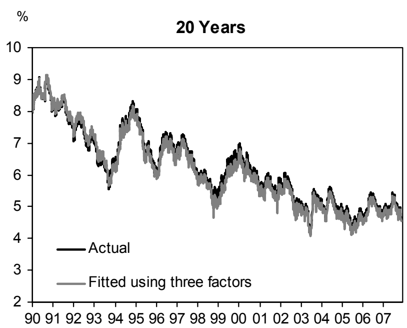

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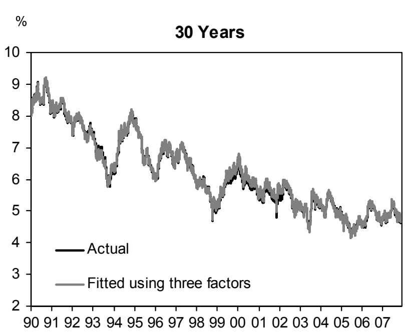
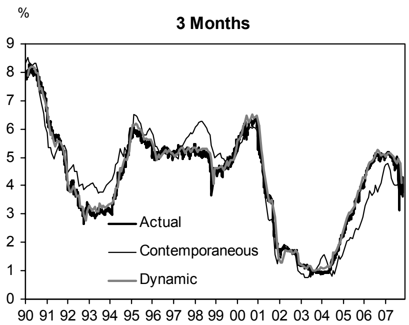
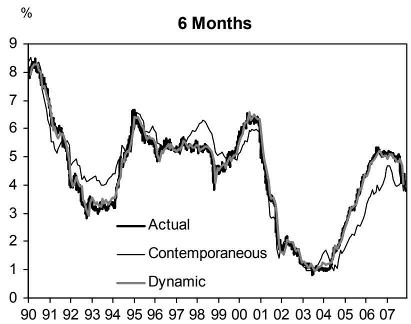


Sources: Haver Analytics, Goldman Sachs calculations.

Fair Values of Interest Rates at Different Maturities



Sources: Haver Analytics, Goldman Sachs calculations.



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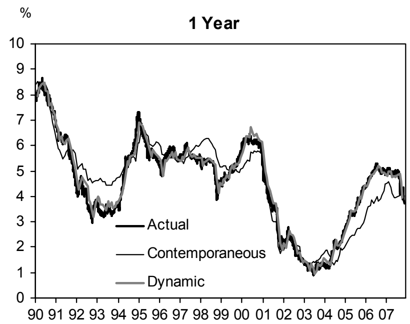

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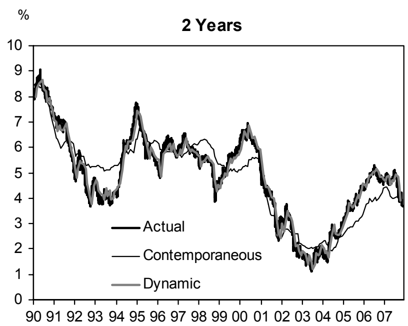

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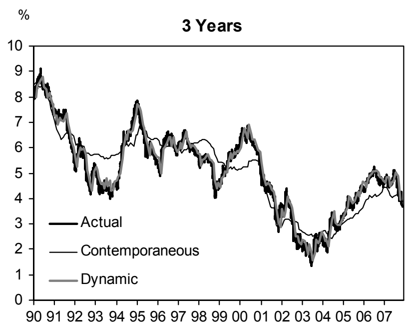
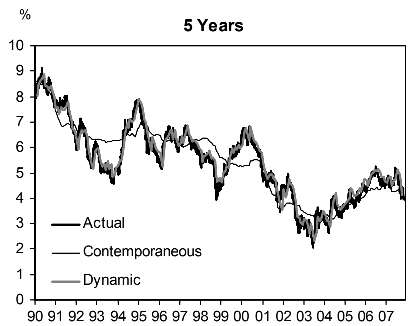


Sources: Haver Analytics, Goldman Sachs calculations.



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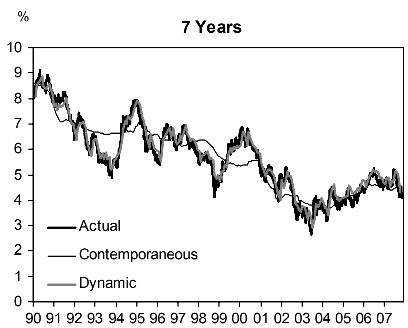

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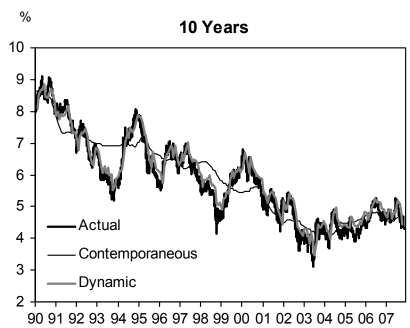
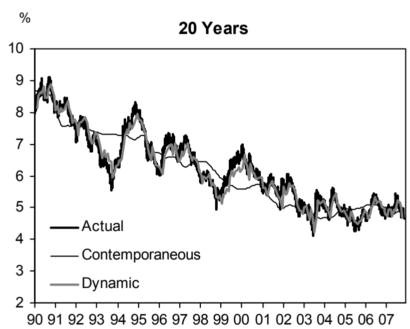


Sources: Haver Analytics, Goldman Sachs calculations.



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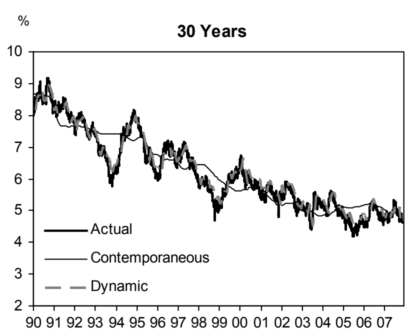
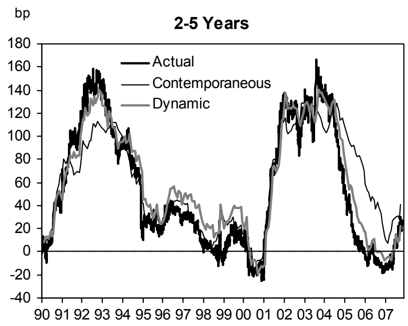
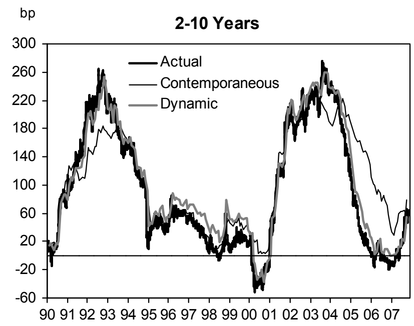


Sources: Haver Analytics, Goldman Sachs calculations.

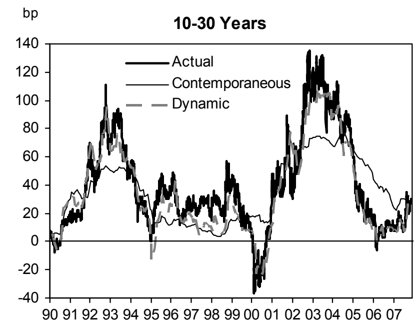
Fair Values of Interest Rate Spreads at Different Maturities



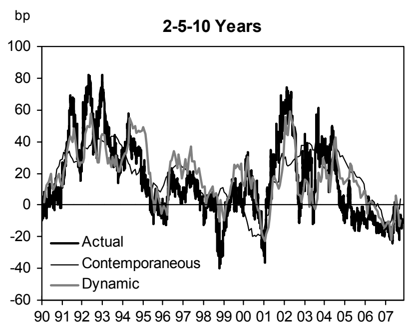
Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.



Sources: Haver Analytics, Goldman Sachs calculations.



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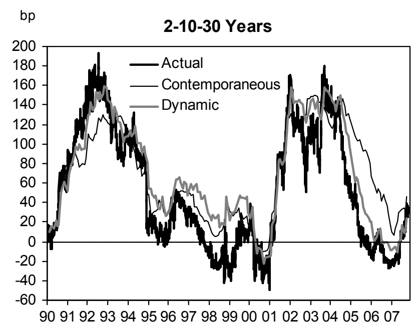


Sources: Haver Analytics, Goldman Sachs calculations.

Box 1: Model of the Term Structure of Interest Rates

Our approach builds on two ideas – that the whole term structure of interest rates can be described reasonably well by only three latent factors (level, slope and curvature), and that these three factors can be related to specific economic variables (Federal Funds rate, and expectations of growth and inflation at different horizons).

To begin with, we use the variation of the Nelson-Siegel exponential components framework suggested by Diebold and Li (2006)* to distil the yield curve into three time-varying parameters, which can be conveniently interpreted as the level, slope and curvature factors. Specifically, the yield curve is modelled as:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right),$$

where for each date t

$y_t(\tau)$ is a yield of a bond with the maturity of τ months,

λ_t is a parameter governing the exponential decay rate,

$\beta_{1,t}$ is a parameter responsible for the characteristics of a yield curve at long-term maturities (high τ),

$\beta_{2,t}$ is a parameter responsible for the characteristics of a yield curve at short-term maturities (low τ),

$\beta_{3,t}$ is a parameter responsible for the characteristics of a yield curve at medium-term maturities (intermediate values of τ).

Following Diebold and Li (2006), we fix λ_t at 0.05 in order to have the loading on a medium-term parameter $\beta_{3,t}$ maximised at exactly 3 years (which is different from the original authors' choice of 0.0609 maximising the loading on $\beta_{3,t}$ at 2.5 years), and then estimate $\beta_{1,t}$, $\beta_{2,t}$, and $\beta_{3,t}$, viewing them as latent dynamic factors determining, respectively, level, slope and curvature of a yield curve. For each time period, parameters are estimated simultaneously by OLS using the cross-section of observed yields. We use the Federal Reserve daily frequency data for constant maturity US Treasury yields (3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 20 years, and 30 years), starting in January 1990 and ending in November 2007.

As a result, we obtain the time series of three factors, and transform the task of explaining interest rates at each maturity directly into one of explaining only these three common factors, which in turn can be easily translated into interest rates at various maturities (thus reducing the dimensionality of our problem). Originally, Diebold and Li (2006) model three factors as univariate AR(1) processes. With the aim of improving the explanatory and forecasting power of the model, as well as in order to gain a better understanding of fundamental factors that determine the term structure and drive the dynamics of interest rates, we take the model a step further.

Specifically, we augment it by economic variables reflecting current monetary policy stance and expected macroeconomic environment, using simple economic logic to impose a little more structure on the model. Thus, the nominal interest rate can be viewed as a combination of real rates and inflation (and a random risk premium process), which in turn can be related to expected GDP growth and CPI inflation, respectively. Then, we postulate that the level factor, which pins down the long end of the yield curve, is largely determined by long-term expectations of growth and inflation. Similarly, the slope factor, which is relatively more important for the short-term interest rates, is dependent on short-term expectations of growth and inflation. In the same fashion, the curvature factor, which has the highest loading in the intermediate part of the yield curve, is assumed to be a function of medium-term expectations. In other words, we link the term structure of interest rates to the term structure of macroeconomic expectations using the Diebold and Li (2006) model as a bridge. On top of this, the Federal Funds rate is added as an additional explanatory variable for the slope factor, given its importance for the short-term money market. Lastly, to allow for the persistence of the three factors, their 1-calendar-month lags are included as well. This gives us the following set of equations:

$$\beta_{L,t} = \delta_{L,0} + \delta_{L,1}\beta_{L,t-20} + \delta_{L,2}g_{L,t} + \delta_{L,3}i_{L,t} + \varepsilon_{L,t}$$

$$\beta_{S,t} = \delta_{S,0} + \delta_{S,1}\beta_{S,t-20} + \delta_{S,2}g_{S,t} + \delta_{S,3}i_{S,t} + \delta_{S,4}f_t + \varepsilon_{S,t}$$

$$\beta_{M,t} = \delta_{M,0} + \delta_{M,1}\beta_{M,t-20} + \delta_{M,2}g_{M,t} + \delta_{M,3}i_{M,t} + \varepsilon_{M,t},$$

where

$\beta_{i,t-20}$ are 20-business days lags of $\beta_{i,t}$,

$g_{L,t}$ is GDP growth expectation 6-10 years ahead,

$i_{L,t}$ is CPI inflation expectation 6-10 years ahead,

$g_{S,t}$ is GDP growth expectation 1 year ahead,

$i_{S,t}$ is CPI inflation expectation 1 year ahead,

$g_{M,t}$ is GDP growth expectation 3 years ahead,

$i_{M,t}$ is CPI inflation expectation 3 years ahead,

* Diebold, F.X., and Li, C. 2006. "Forecasting the Term Structure of Government Bond Yields." *Journal of Econometrics*, 130, 337-364.

Box 1 continued…

f_t is Federal Funds target rate,

$\delta_{i,j}$ are parameters,

$\varepsilon_{i,t}$ are regression disturbance terms.

In addition to this mainstream formulation, which includes as explanatory variables the lags of the three factors, we also define its restricted version, which omits the lags and only uses contemporaneous economic variables as regressors (i.e., parameters $\delta_{i,1}$ are set equal to 0 there).

Data on US macroeconomic expectations are sourced from Consensus Economics. In particular, we have monthly data on short-term expectations for the end of current (T) and subsequent ($T+1$) calendar years. Using data on short-term expectations with moving horizons, we calculate approximate 1-year-ahead expectations simply as geometric moving averages. Also, we have bi-annual data on medium-term expectations for the end of years $T+2$ and $T+3$, which allows us to calculate approximate 3-years-ahead expectations in the same way as above. Lastly, we have bi-annual data on long-term (average of 6 to 10 years ahead) expectations. Merging these data sets, we can estimate the parameters $\delta_{i,j}$ for each equation separately by simple OLS. Using the estimated parameters, we are able to calculate the fitted values of level, slope and curvature factors as implied by economic fundamentals. The frequency of such fitted values will be monthly for the slope and bi-annual for the level and curvature factors.

However, any serious practical use requires us to have output of higher than bi-annual frequency. Box 2 describes how the problem of survey data deficiency is tackled and monthly-frequency estimates of medium (3 years ahead) and long-term (6-10 years ahead) macroeconomic expectations are obtained. With such synthetic measures of longer-term expectations at hand, and using the parameters estimated above, we can calculate the fitted values of level, slope and curvature factors at monthly frequency.

(Note that our inputs and outputs here have uneven frequency, because Consensus Economics data are released at varying calendar dates, and the dating of daily-frequency variables such as $\beta_{i,t}$, $\beta_{i,t-20}$, and f_t have to be matched with the dates of expectations data releases.)

Finally, translating these three fitted factors into implied term structure of bond yields gives what we call a 'fair value' yield curve. Depending on the manner of calculating the values of fitted factors – whether the lags of the three factors were included or not – we separate 'dynamic' and 'contemporaneous' fair value yield curves.

Detailed results of estimation and diagnostics are available upon request.

Box 2: Synthetic Measures of Longer-Term Macroeconomic Expectations

The main idea behind our approach is that useful information about latent dynamics of medium- (3 years ahead) and long-term (6-10 years ahead) macroeconomic expectations can be extracted from observed changes in short-term (1 year ahead) expectations.

Specifically, we construct a state-space system that assumes that each longer-term expectations variable is a function of its own lag, short-term expectations of growth and inflation, and a random shock. Furthermore, these longer-term expectations variables are realised monthly, but observed only bi-annually. As a result, we obtain a set of four state equations determining the dynamics of underlying unobserved longer-term expectations:

$$\gamma_{L,t} = \phi_{L,0} + \phi_{L,1}\gamma_{L,t-1} + \phi_{L,2}g_{S,t} + \phi_{L,3}i_{S,t} + \xi_{L,t}$$

$$\gamma_{M,t} = \phi_{M,0} + \phi_{M,1}\gamma_{M,t-1} + \phi_{M,2}g_{S,t} + \phi_{M,3}i_{S,t} + \xi_{M,t}$$

$$\iota_{L,t} = \varphi_{L,0} + \varphi_{L,1}\iota_{L,t-1} + \varphi_{L,2}g_{S,t} + \varphi_{L,3}i_{S,t} + \zeta_{L,t}$$

$$\iota_{M,t} = \varphi_{M,0} + \varphi_{M,1}\iota_{M,t-1} + \varphi_{M,2}g_{S,t} + \varphi_{M,3}i_{S,t} + \zeta_{M,t},$$

where:

$\gamma_{L,t}$ is unobserved GDP growth expectation 6-10 years ahead,

$\iota_{L,t}$ is unobserved CPI inflation expectation 6-10 years ahead,

$\gamma_{S,t}$ is unobserved GDP growth expectation 1 year ahead,

$\iota_{S,t}$ is unobserved CPI inflation expectation 1 year ahead,

$\gamma_{M,t}$ is unobserved GDP growth expectation 3 years ahead,

$\iota_{M,t}$ is unobserved CPI inflation expectation 3 years ahead,

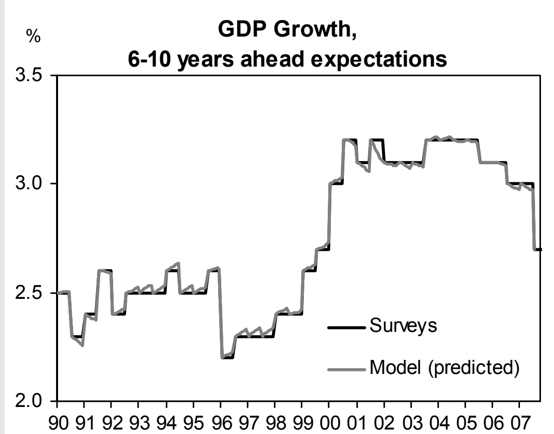
$\phi_{i,j}$ are parameters,

$\varphi_{i,j}$ are parameters,

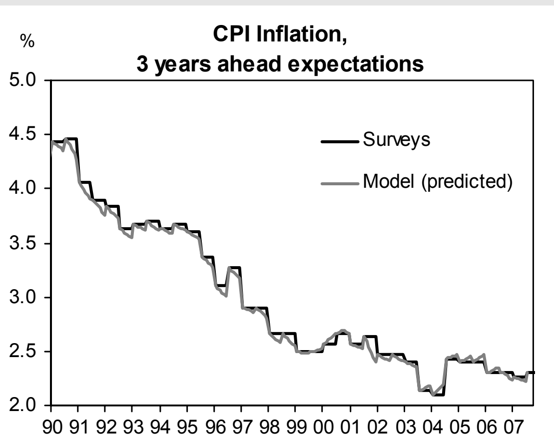
$\xi_{i,t}$ are regression disturbance terms,

$\zeta_{i,t}$ are regression disturbance terms.

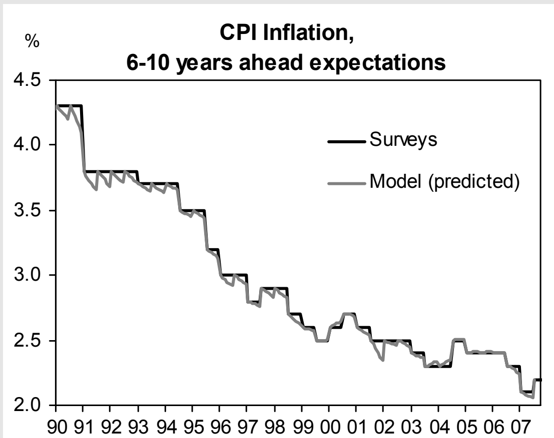
Sources: Consensus Economics, Goldman Sachs calculations.



Sources: Consensus Economics, Goldman Sachs calculations.



Sources: Consensus Economics, Goldman Sachs calculations.



Sources: Consensus Economics, Goldman Sachs calculations.

Box 2 continued…

At the same time, we have a set of four observation equations relating observed longer-term expectations to underlying unobserved expectations variables:

$$g_{L,t} = \gamma_{L,t}$$

$$g_{M,t} = \gamma_{M,t}$$

$$i_{L,t} = \iota_{L,t}$$

$$i_{M,t} = \iota_{M,t} .$$

Obviously, given that longer-term expectations $g_{L,t}$, $g_{M,t}$, $i_{L,t}$, and $i_{M,t}$ are observed only bi-annually, the left-hand side variables in the last 4 equations are missing and need to be estimated. To calculate the forecasts and smoothed estimates of unobserved variables, we use the Kalman filter algorithm, and with its help, the whole system is estimated by the method of maximum likelihood.

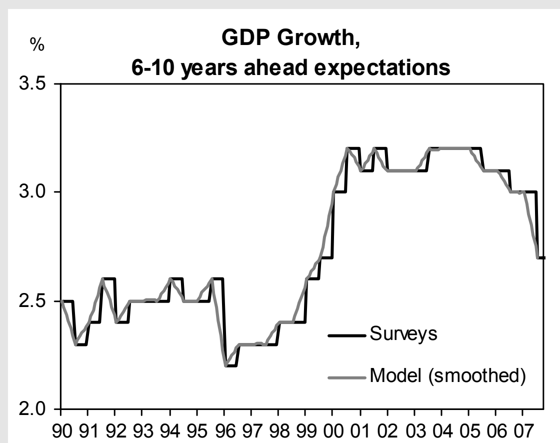
Essentially, between the (infrequent) releases of longer-term expectations surveys, the model updates its estimates of current longer-term expectations with new information contained in the (more frequent) releases of short-term expectations reports. Four charts on page 55 juxtapose the model's 1-step-ahead forecasts of longer-term consensus expectations against Consensus Economics' actual survey data. Visually, it becomes evident that the model's predictions usually do anticipate the direction of future changes in reported consensus expectations.

The charts on the previous page plot model estimates of consensus expectations based only on information physically available at each point in time. *Ex post*, it is possible to recalculate past estimates using all the new information available at the present time. Such smoothed estimates using currently available information are graphed in the four charts below.

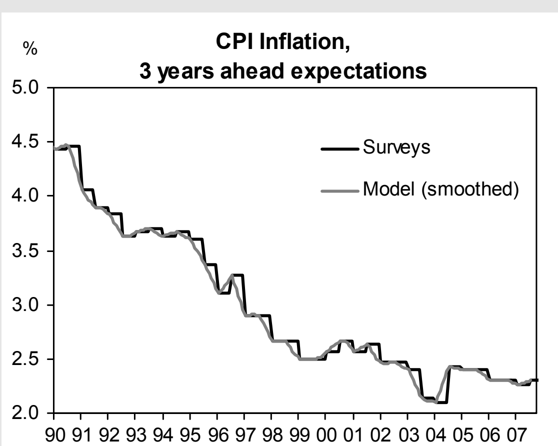
While the former set of charts demonstrates the predictive performance of the model, the latter set depicts the model's best estimates of unobserved monthly-frequency consensus expectations series, which in turn can serve as inputs for other uses.

Detailed results of estimation and diagnostics are available upon request.

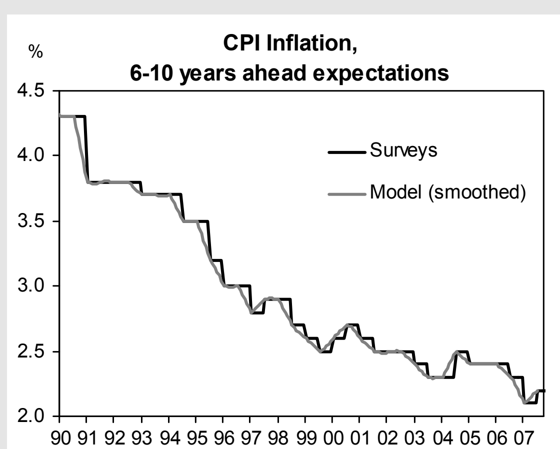
Sources: Consensus Economics, Goldman Sachs calculations.



Sources: Consensus Economics, Goldman Sachs calculations.



Sources: Consensus Economics, Goldman Sachs calculations.



Sources: Consensus Economics, Goldman Sachs calculations.