Sudoku Gets Bigger and Forward-Looking!

1. Overview

Our ‘Sudoku’ bond valuation model, presented in early 2006, provides a framework to estimate the ‘fair value’ of 10-year government bond yields in advanced economies based on three macroeconomic factors (short rates, activity growth and inflation) and accounting for contemporaneous spill-over effects across markets.

In this Viewpoint, we introduce an expanded and slightly modified version of the model. To begin with, the number of countries entering Sudoku increases from four to seven: in addition to the US, Germany, Japan, and the UK, the framework is extended to Canada, Switzerland, and Sweden.

Moreover, in place of current values for the three macro factors, the model now feeds off 1-year-ahead consensus expectations for these same variables. This allows to better capture the forward-looking nature of asset prices.

Our latest estimates indicate that global bonds are still expensive. Once credit markets jitters eventually subside, 10-yr yields will likely face renewed upward pressures. Unlike earlier this year, the degree of overvaluation is not enough to justify an aggressive short trading stance, however.

Within our enlarged universe, Swiss 10-yr bonds are the ones offering presently most value, while Canada sits at the other end of the valuation spectrum. The recent sell-off has acted to reduce valuation differences between G4 markets, and hence the scope to express convergence trades across them.

Our economists’ forecasts imply that Swiss bonds, followed at some distance by US Treasuries, offer the most attractive expected returns relative to the forwards by the end of the year. On the other hand, Swedish bonds are the worst placed.
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2. An (Extended) Sudoku Primer
To remind readers, Sudoku is based on a system of equations for government bond yields in which any given country X’s 10-yr rates takes the following form:

\[ LR_X = \beta_{X,0} + \beta_{X,1} \times GR_X + \beta_{X,2} \times IN_X + \beta_{X,3} \times SR_X + \beta_{X,4} \times LR^*_X + \varepsilon_X \]

where:

- \( LR_X \) is the 10-year bond yield in country X,
- \( GR_X \) is the 1-year ahead expectation of year-on-year GDP growth rate in country X,
- \( IN_X \) is the 1-year ahead expectation of year-on-year CPI inflation rate in country X,
- \( SR_X \) is the 1-year ahead expectation of the 3-month interest rate in country X,
- \( LR^*_X \) is a global bond yield variable, specific to country X and constructed as the weighted-average of foreign long rates, using trade shares as weights:

\[ LR^*_X = \omega_{X,A} \times LR_A + \omega_{X,B} \times LR_B + \ldots \]

where \( \omega_{X,A} \) represents the share of country A in country X’s trade volume.

Finally, \( \varepsilon_X \) is an error term for country X.

The sample starts in January 1990, at a monthly frequency. Data on expectations are either coming directly from or calculated using Consensus Economics survey reports. Our estimation approach is described in the 2006 issue of The FX Market Annual, Chapter 12.

3. Current Solutions to the Bond Puzzle
The charts on page 5 plot historical ‘fair values’ for 10-year yields of major government bonds against actual values. As can be seen, both bond yields and their fitted values in all seven countries included in our universe tend to co-move. Currently, in all markets bond yields lie below our measure of ‘fair value’ based on the three macro economic factors entering the framework (and accounting for cross-country spill-overs).

The model’s output can also be summarized in terms of standard deviations of mis-valuation. This allows constructing a statistically meaningful ranking of ‘value’ in the different bond markets at any given point in time.

As illustrated in the charts on page 1, yields in all major bond markets have undershot their equilibrium levels since early 2005. Coming into this year, US Treasuries, JGBs and German Bunds were meaningfully expensive.

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1. We are aware that the large swing in fiscal positions from deficit to surplus in Sweden and Canada over our sample period may have had an impact on the respective equilibrium long rates. Reassuringly, our analysis suggests that these effects do not meaningfully change current baseline results.
Currently, most markets are close to one standard deviation expensive – hence not enough to justify an aggressive short stance. The cheapest market in our universe is Switzerland, while the most expensive is Canada.

4. Drilling Down Further

Sudoku can be employed to decompose changes in equilibrium yields into contributions from different macro factors, both retrospectively and as projected into the future. One can also isolate the basis point value of the error-correction process built into the model when there are deviations from equilibrium. The table for the US Treasuries below provides an illustration.

The spot level of Treasury yields, 4.99%, is shown in the top right side of the table. This compares with a ‘fair value’ estimate of 5.71%, based on the consensus expectations of short rates, growth and inflation in 12-months period from now [see column labelled Total (CE)].

Our assumptions on future macroeconomic developments may well differ from market average expectations. For example, we are currently of the view that US growth will slow, Japanese inflation increase, and the ECB hikes rates more than implied by the forwards. Based on these forecasts, we can build a separate estimate of 10-yr ‘fair value’ consistent with our views. As shown at the top of the fourth column from the right in the table, the current ‘fair value’ based on GS inputs is 5.90%, higher than consensus.

Finally, we can use the framework to project where actual 10-yr yields may be heading over the coming three months based on the combination of two inter-related forces: the bridging of the valuation gap and the projected change in equilibrium itself as the macro landscape evolves according to our baseline projections. The column labelled ‘mis-valuation’ captures the error-correction dynamics (worth a cumulative +27bp over the next 3-mths) while the remaining cells attribute the projected change in ‘fair value’ (+3bp) to the different local and global macro drivers.

These results are presented graphically on the charts on pages 2-3. As can be seen, by the end of 2007, the least attractive bonds relative to forwards are those in Sweden and Japan, while Swiss and US ones look the most attractive.

5. Conclusions

The reliance on a parsimonious set of three macro factors is both Sudoku’s strength and limitation. The risks of
over-fitting are minimized and yet we can quantify the impact of movements in any of these factors in one country onto long rates both locally and abroad. The weakness of the approach lies in the fact that linear framework based on just three macro inputs will miss more transitory influences on long-yields. As a support to the formulation of strategy, we have chosen to have a coarser but more stable framework, and to use judgement or auxiliary regression analysis to account for deviations from equilibrium.

An important advantage of the model lies in its global equilibrium structure, whereby the pricing of long-rates in any one country depends not only on domestic macro underpinnings, but also on the pricing of long rates abroad. As a result, Sudoku ‘fair values’ estimates are consistent with each other. This allows us to make judgements both on valuation of government bonds in relation to each other, and on absolute mis-valuation of bonds in individual countries.

The latest message from Sudoku is threefold:

■ Investors should preserve a mild negative bias on the direction of bond yields.

■ The opportunities to capitalize on inter-country valuation discrepancies have declined after the recent sell-off. Currently, Canada and Switzerland sit on opposite sides of the value spectrum.

■ In relation to the forwards, Swiss and, at some distance, US 10-yr rates are the most attractive on a 3-6 months investment horizon, while Swedish and Japanese bonds the least.

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Canada, 10-yr Bond Yield

Switzerland, 10-yr Bond Yield

Japan, 10-yr Bond Yield

UK, 10-yr Bond Yield

USA, 10-yr Bond Yield

Germany, 10-yr Bond Yield

Sweden, 10-yr Bond Yield
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